



Introduction

One of the best known methods for detecting period variation mechanisms in eclipsing binaries is based on the detection and analysis of the eclipse timing variations (ETV). The apparent cyclical period variation of an eclipsing binary can provide an indirect evidence of a circumbinary companion as a result of gravitational attraction (Light-Time Effect, LITE; Irwin 1952) or might have as a cause stellar magnetic activity (Applegate 1992).

The formation of binary systems, especially evolved close binary stars, their evolutionary scenarios as well the evolution of planetary systems can be explored and understood by the investigation of circumbinary objects (Zinnecker 2002, Schleicher et al. 2015, Toonen et al. 2016).

Here we present an extensive analysis of O-C (Observed-Calculated times of minima) diagrams for the over-contact (W Uma type) TZ Boo binary system and the post-common envelope binaries (PCEB) NSVS 142568125, NSVS 07826147 by applying a series of optimization techniques as the best strategy in the determination of the best fitting curve, since the topology of parameter space arises difficulties in the determination of the global (lowest χ^2 value) solution (Kallrath and Milone 2009).

O-C diagram analysis

We constructed O-C diagrams with archival data and using new times of minima of photometric observations carried out at Mythodea Observatory at the University of Patras during the years 2014-2017 with a 35.5 cm f/6.3 Schmidt-Cassegrain telescope.

The strategy of finding the best fitting curve consists of a first determination using the Nelder-Mead Downhill Simplex and Levenberg-Marquardt algorithms, while the globality of solution is sought by a Heuristic Scanning scheme which implements the two aforementioned methods with a parameter kicking or by a Genetic Algorithm (PIKAIA, Charbonneau 1995). As a last step of scanning the topology of χ^2 parameter space and in order to acquire more objective parameter value errors we implement an M-H MCMC algorithm.

All these methods are implemented using the TIMING RESIDUALS & LITE package of Christopoulou et al. (2015).

TZ Boo (HIP 74061, $V=10.45$) is one of the most interesting and unusual W Uma binaries because it has exhibited several period changes since its discovery, its light curve type switches between A and W types and its spectroscopic study (Pribulla et al. 2009) revealed a third component which did not confirmed though by an adaptive optics search (Rucinski et al. 2007).

Using a collection of data spanning 72 years (1948-2019) we confirmed the spectroscopically detected circumbinary object of stellar mass $M_3 = 0.84$ Msun (for coplanar orbit) and revealed the existence of another companion with mass $M_4 = 0.2$ Msun.

The orbital parameters are shown in Table 1 while the best fitting model is shown in Fig.1 (left) for the third body and a second companion's solution is presented on the residuals of Fig.1 (right).

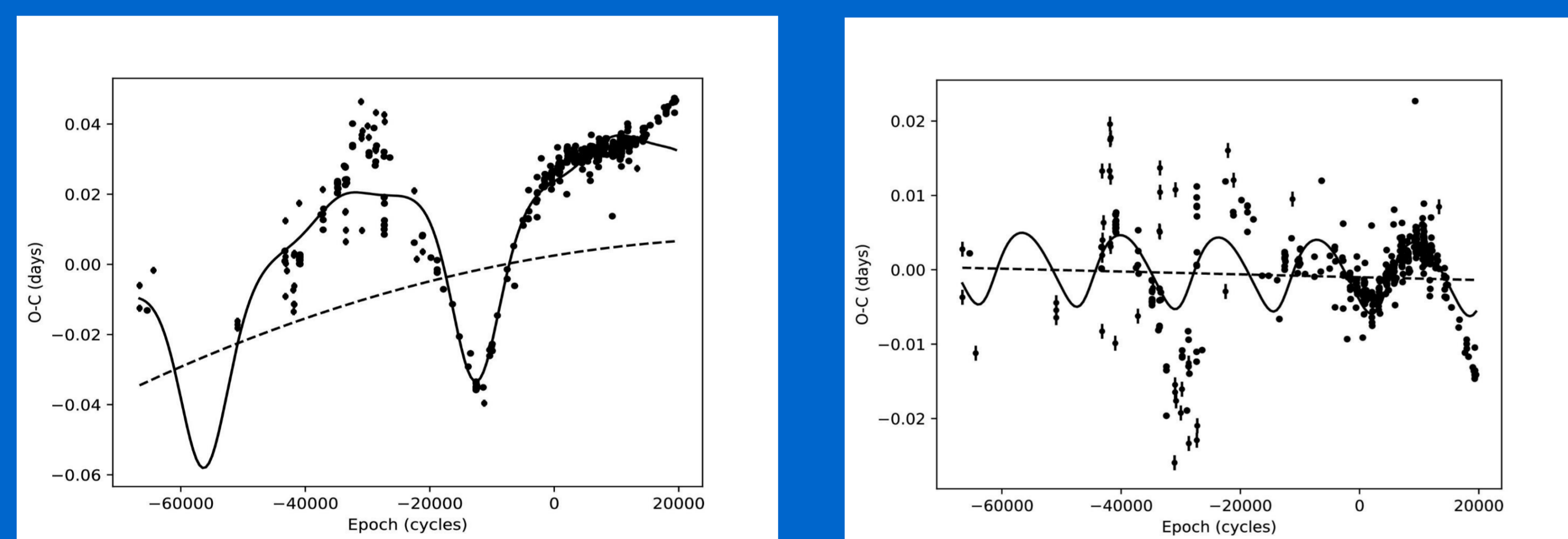


Fig. 1: Best fitting model for the tertiary (left) and second (right) companion of TZ Boo according to LITE parameters of table 1.

Parameter	M-H MCMC		Genetic Algorithm	
	Value	Error	Value	Error
e_3	0.79	± 0.02	0.81	± 0.01
A (days)	0.021	± 0.001	0.0182	± 0.0003
ω_3 (rad)	6.03	± 0.01	6.01	± 0.02
$qmt = dP/2aE$ (days cycle ⁻¹)	-0.051×10^{-10}	$\pm 0.009 \times 10^{-10}$	-0.029×10^{-10}	$\pm 0.004 \times 10^{-10}$
P_3 (years)	35.51	± 0.11	36.76	± 0.24
T_3 (HJD)	2448659.116	± 0.01	2448713.996	± 45.43
$P_{bin.}$ (days)	0.2971597	± 0.00	0.2971594	± 0.00
T_0 (HJD)	2452500.1577	± 0.0007	2452500.156	± 0.00

Simplex-LM		
Parameter	Value	Error
e_4	0.28	± 0.10
A (days)	0.0049	± 0.0002
ω_4 (rad)	0.73	± 0.36
P_4 (years)	13.41	± 0.11
T_4 (HJD)	2453515.0274	± 270.189

Table 1: Orbital parameters of two companions model for the TZ Boo system. The stability and globality of first companion's solution confirmed with Heuristic Scanning and Genetic algorithm implementation, whereas the parameters of second companion resulted only from Simplex and Levenberg-Marquardt algorithms.

References

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NSVS 14256825 (V* V1828 Aql) is a PCEB of HW Vir (sdOB+dM) type (Almeida et al. 2012) whose period variation has been studied by many investigators with the latest including those of Nasiroglu et al. (2017) and Pulley et al. (2018).

Our period analysis was based on all previous published times of minima and those acquired at Mythodea Observatory covering a total 11 years time span (2007-2017). The apparent cyclic behavior of O-C eclipse timings are attributed to a Jovian-type circumbinary planet of mass $M_3 = 16$ MJup. and orbital elements $e_3 = 0.19$, $P_3 = 10$ years. This one companion LITE model is in agreement with the latest published period investigations.

In Table 2 we present the model's parameter values according to Heuristic Scanning and M-H MCMC algorithms, while in Fig.2 (left) and Fig.3 the best fitting LITE model and posterior distributions as they resulted from the M-H MCMC χ^2 topology exploration.

Parameter	Heuristic Scanning		M-H MCMC	
	Value	Error	Value	Error
e_3	0.19	$\pm 4.25 \times 10^{-9}$	0.191	± 0.002
A (days)	0.00058	$\pm 1.07 \times 10^{-12}$	0.001	± 0.00
ω_3 (rad)	2.92	$\pm 1.49 \times 10^{-8}$	2.92	± 0.01
P_3 (years)	10.38	$\pm 3.41 \times 10^{-8}$	10.38	± 0.12
T_3 (HJD)	2456358.946	$\pm 8.54 \times 10^{-6}$	2456358.946	± 0.01
$P_{bin.}$ (days)	0.1103742	± 0.00	0.1103742	± 0.00
T_0 (HJD)	2454274.21	± 0.00	2454274.2085	± 0.00

Table 2: One companion's LITE model parameters for NSVS 14256825.

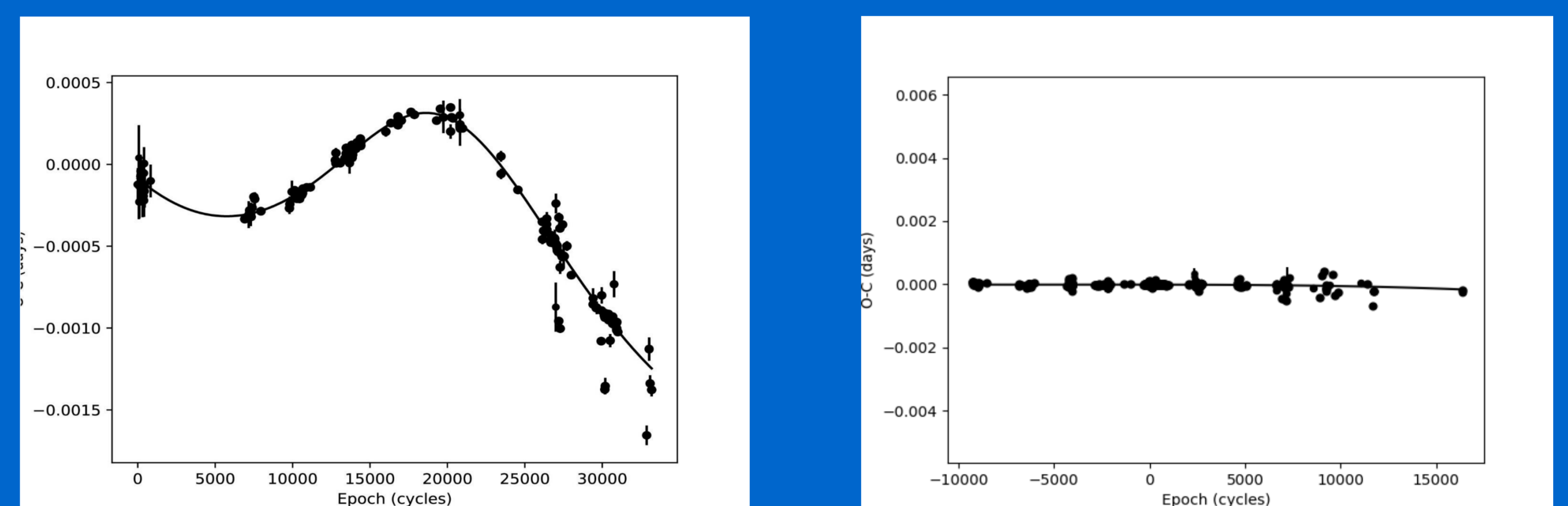


Fig. 2 (Left) Best fitting curve for a Jovian-type circumbinary object revolving around NSVS 14256825. (Right) O-C diagram of NSVS 07826147 with a linear fit.

NSVS 07826147 (2M 1533+3759, $V=13.08$) is a PCEB of HW Vir (sdB+dM) type whose detailed absolute parameters derived by the combination of photometric and spectroscopic observations were published by For et al. (2010). The latest period variation analysis of Lee et al. (2017) and Pulley et al. (2018) did not reveal a variation during a time span of 14 years (2004-2017) and the same is confirmed here for an expanded times span of 16 years (2004-2019). However, a downward trend is apparent in data of the last two years, therefore a future monitoring of the system is required.

Applegate investigation

The Applegate mechanism is investigated as a possible alternative scenario in TZ Boo and NSVS 14256825 cyclical variation cases according to *thin shell model* (Applegate 1992, Tian et al. 2009) and the more accurate *two zone model* (Volschow et al. 2016).

Star	$\Delta E/E_{sec}$ ($\Omega_{dr} = 0$)	$\Delta E/E_{sec}$	$\Delta E/E_{sec}$
	Applegate (1992)	Tian et al. (2009)	Volschow et al. (2016)
	Thin shell model	Thin shell model	Two zone model
NSVS 14256825	11.92	9.52	180.62
TZ Boo	0.12	0.1	-

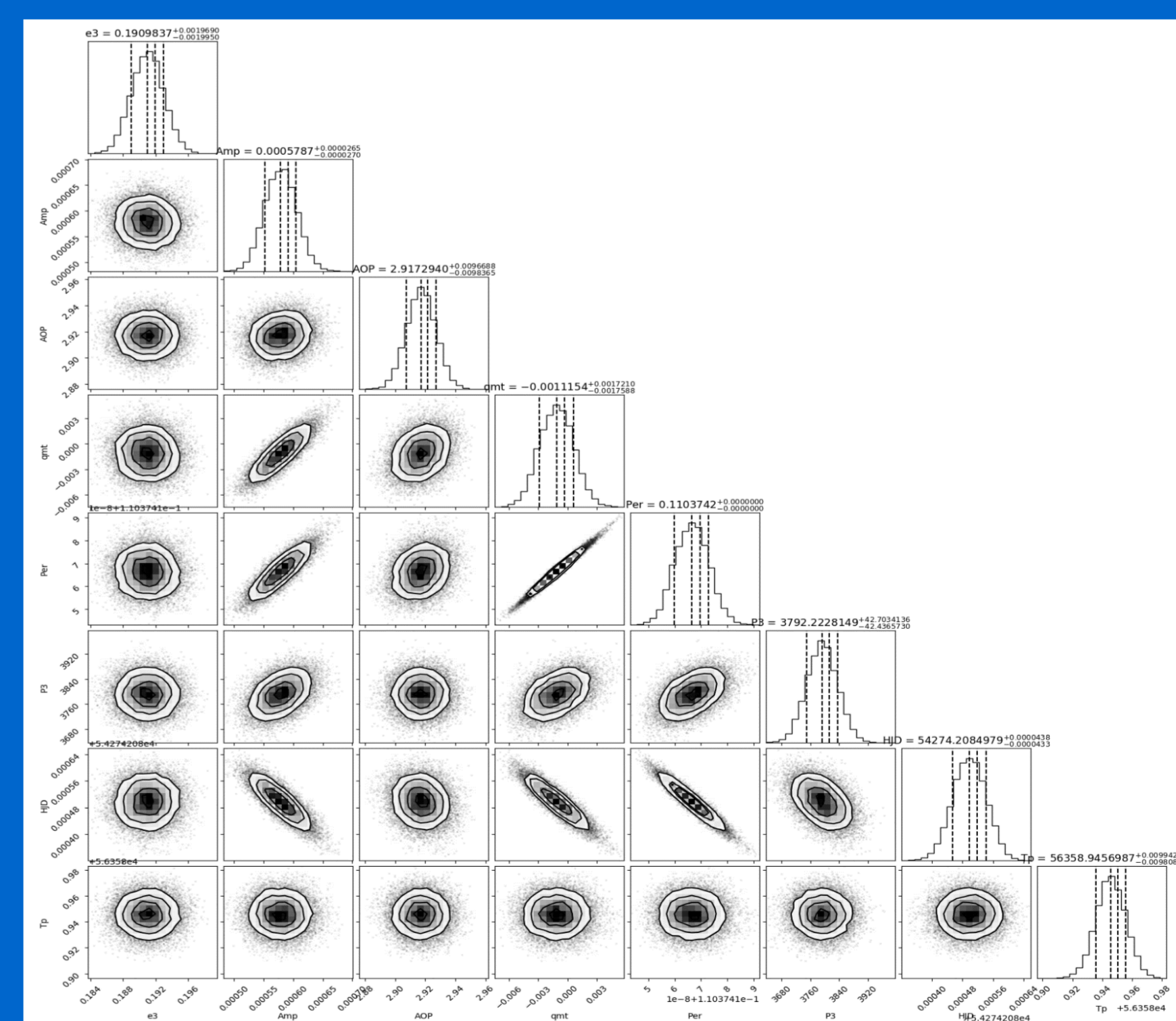


Fig. 3 Posterior distributions and density plots for NSVS 14256825 as resulted from M-H MCMC.

Conclusions

The extensive period variation analysis of O-C eclipse timings diagrams indicated a stable solution of one sub-stellar companion around NSVS 14256825 and a stable stellar companion solution around TZ Boo. The residuals of TZ Boo's one companion LITE model reveal the possible existence of a second circumbinary object, however, the energy threshold according to Applegate mechanism is sufficient to explain the cyclical variation, therefore a contribution of the mechanism in residuals can be taken into account.

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